Letter

Nonequilibrium critical dynamics with emergent supersymmetry

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Proposed as an elegant symmetry relating bosons and fermions, spacetime supersymmetry (SUSY) has been actively pursued in both particle physics and emergent phenomena in quantum critical points (QCPs) of topological quantum materials. However, how SUSY casts the light on nonequilibrium dynamics remains open. In this Letter, we investigate the Kibble-Zurek dynamics across a QCP with emergent $\mathcal{N} = 2$ spacetime SUSY between the Dirac semimetal and a superconductor through a large-scale quantum Monte Carlo simulation. The scaling behaviors in the whole driven process are uncovered to satisfy the full finite-time scaling (FTS) forms. More crucially, we demonstrate that the emergent SUSY manifests in the intimate relation between the FTS behaviors of fermionic and bosonic observables, namely the fermions and bosons acquire identical anomalous dimensions. Our work not only brings a fundamental ingredient into the critical theory with SUSY, but also provides the theoretical guidance to experimentally detect QCPs with emergent SUSY from the perspectives of the Kibble-Zurek mechanism and FTS.

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Introduction. Supersymmetry (SUSY), as a symmetry that interchanges bosonic and fermionic fields with each other, was proposed to ingeniously solve the hierarchy problem by exactly canceling the fermion and boson loop corrections to the mass of the Higgs particle [1–7]. Although the partner particles predicted by SUSY are still awaiting experimental verification, the elegance of SUSY theory has extensively impacted modern physics from high-energy physics to condensed matter physics [8–30]. For example, the invariance under local SUSY transformations can automatically reproduce Einstein's general relativity, resulting in the theory of supergravity [2,6]. In addition, SUSY and its spontaneous breaking were proposed in both disordered and chaotic systems [28–32]. Moreover, recently, it was theoretically demonstrated that spacetime SUSY can spontaneously emerge at some quantum critical points (QCPs) in quantum many-body systems [8–27]. For instance, a (2+1)-dimensional [(2+1)D] spacetime SUSY is proposed to emerge at the superconducting QCP [23], paving another route to experimentally realize emergent SUSY.

Up to now, SUSY is generally investigated as an equilibrium property of the ground states. However, it is still unclear how SUSY affects the nonequilibrium dynamics, wherein both the ground state and excited states are involved [33–36]. This question is motivated by the fact that nonequilibrium dynamics is ubiquitous in nature from the inflation of the Universe on the cosmoscopic scale to the collisions of particles in the Large Hadron Collider on the subatomic scale. In particular, near the critical point, nonequilibrium processes show remarkable universal time-dependent scaling behaviors dictated by the divergent correlation timescale [33,34,37]. In recent years, programmable quantum devices have been developed as practical and tunable platforms to realize QCPs [27,38–40], in which nonequilibrium dynamics is naturally present and utilized to detect quantum critical properties. Hence, unraveling the nonequilibrium dynamics of the QCP with emergent SUSY will definitely enrich the theory of supersymmetric quantum criticality and shed light on the experimental observation of emergent spacetimexd SUSY at QCPs.

Among various nonequilibrium realizations, the celebrated Kibble-Zurek mechanism (KZM) attracts special attention [41,42]. Providing a unified description on the generation and dynamic scaling of topological defects after the linear quench across a critical point [41,42], the KZM has been extensively investigated in the cosmological phase transition, and classical and quantum phase transitions, from both theoretical and experimental sides [41–67]. Moreover, dynamic critical behaviors for other quantities are found to exist in the whole driven process [68–73]. A finite-time scaling (FTS) theory was proposed with complete scaling forms to understand these scaling properties [74–76]. In addition, the FTS forms have also been verified in experiments and numerical simulations for different driving protocols [38–40,74–85]. Recently, KZM and FTS also show their power in state preparations and the detection of critical properties in fast-developing programmable quantum devices [38–40,84–88], which provides a promising avenue to the experimental realization and

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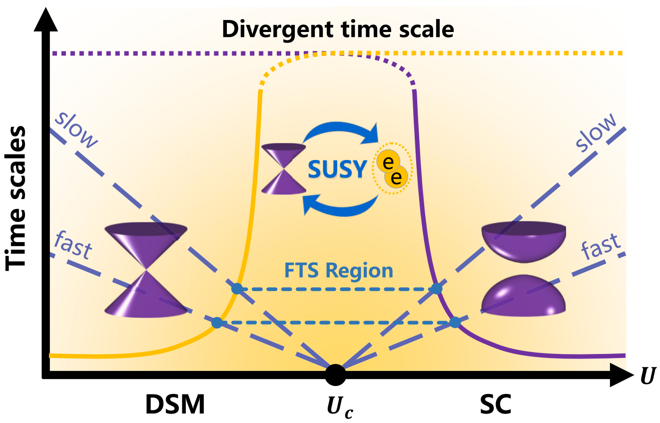

FIG. 1. Sketch of the phase diagram with a SUSY critical point and the protocol for driven dynamics with different initial states. The correlation timescales for both bosons (yellow curve) and fermions (violet curve) are finite in one phase (solid) but divergent (dotted) in the other phase. Far from the critical point, different from the conventional KZM, the transition time (long-dashed line) here is smaller than the correlation time of fermionic degrees of freedom in the DSM phase (bosonic degrees of freedom in the SC phase), but larger than that of bosonic degrees of freedom in the DSM phase (fermionic degrees of freedom in the SC phase). Around the critical point, FTS with emergent SUSY (region marked by the short-dashed line) dominates the scaling behaviors.

detection of QCPs featuring exotic critical properties. Given the fundamental importance of SUSY, it is highly desired to investigate driven critical dynamics in the framework of KZM and FTS for a QCP with emergent SUSY.

In this Letter, we explore the nonequilibrium critical dynamics of a QCP with emergent $\mathcal{N}=2$ SUSY separating a Dirac semimetal (DSM) and a superconducting (SC) phase in a system with Stanford Linear Accelerator Center (SLAC) fermions on a square lattice at half filling [23,89–92], as shown in Fig. 1. By simulating the driven dynamics under a linearly varying interaction strength in the imaginary-time direction via the large-scale determinant quantum Monte Carlo (QMC) method [93,94], we uncover that dynamic scaling behaviors depending on the driving rate satisfy the scaling relation of FTS with the critical exponents dictated by SUSY, owing to the principle that both real- and imaginary-time driven dynamics share the same scaling form characterized by the same exponents [95–98] which has been verified in various systems [83,95–99]. Moreover, we unveil another dynamic scaling relation of the fermion correlation, which combines the information from both the gapless DSM phase and SUSY critical point, generalizing the theory of KZM. Our study fundamentally extends the theory of emergent SUSY to nonequilibrium dynamics which could be realized in programmable quantum processors in the near future [27].

Model and its static criticality. We begin with the model which hosts a QCP with emergent SUSY in a square lattice. The Hamiltonian reads [23]

$$H = \sum_{ij}(t_{\boldsymbol{R}} c_{i\uparrow}^{\dagger} c_{j\downarrow} + \text{H.c.}) - U\sum_{i}\left(n_{i\uparrow}-\frac{1}{2}\right)\left(n_{i\downarrow}-\frac{1}{2}\right), \tag{1}$$

in which $c_{i\sigma}^{\dagger}$ ($c_{j\sigma}$) creates (annihilates) an electron at site $\boldsymbol{r}_i$ with spin $\sigma=\uparrow/\downarrow$, $n_{i\sigma}\equiv c_{i\sigma}^{\dagger}c_{i\sigma}$ is the electron number operator, $t_{\boldsymbol{R}}(\boldsymbol{R}=\boldsymbol{r}_i-\boldsymbol{r}_j)$ is the amplitude of long-range hopping given by $t_{\boldsymbol{R}}=\frac{i(-1)^{R_x}}{\frac{L}{\pi}\sin\frac{\pi R_x}{L}}\delta_{R_y,0}+\frac{(-1)^{R_y}}{\frac{L}{\pi}\sin\frac{\pi R_y}{L}}\delta_{R_x,0}$, and $U>0$ measures the strength of the attractive Hubbard interaction. As shown in Fig. 1, when U is small, the system is in the DSM phase with a single Dirac point at $\mathbf{p}=\mathbf{0}$ (namely the Γ point). Note that this model does not contradict with the fermion-doubling theorem because the hopping here is nonlocal and decaying as $1/r$. In contrast, when U is large, the ground state is in the SC state with singlet pairing. In between the two phases, there is a QCP at $U=U_c\approx 0.83$ (in units of the bandwidth) [23].

Although the microscopic Hamiltonian (1) does not have SUSY, it was shown that an emergent $\mathcal{N}=2$ SUSY appears at the critical point U_c [23]. The most remarkable feature is that anomalous dimensions for both bosonic and fermionic fields obtained via QMC simulations are close to their exact values, $\eta_f=\eta_b=\frac{1}{3}$, where η_f and η_b are the fermion and boson fields' anomalous dimensions, respectively [100]. This equivalence of fermion and boson anomalous dimensions is a hallmark of SUSY. Moreover, the correlation length exponent $\nu=0.87(5)$ is also consistent with the critical field theory with SUSY [101–103]. In the following, we will explore the signature of SUSY in the nonequilibrium driven dynamics.

Dynamic protocol. The quantum KZM focuses on the real-time driven dynamics across a QCP [33,34,47–49]. However, simulating the real-time dynamics in higher-dimensional systems is still extremely difficult. Fortunately, it was shown that the imaginary-time driven critical dynamics shares the same scaling forms and critical exponents with the real-time case, except for the detailed scaling functions [95–97]. The reason is that for both cases, when the driving rate is small, the nonequilibrium dynamics of the system is controlled by the low-lying energy excited states which hold critical properties dominated by the QCP. Accordingly, as the only tuning parameter characterizing the extent of departure from the equilibrium state, the driving rate provides a natural characteristic quantity to describe the nonequilibrium dynamic scaling behaviors in both real-time and imaginary-time directions [95–97]. Scaling analyses shows that for both real- and imaginary-time driven dynamics, the driving rate has the same critical dimension. Consequently, one can make a detour to detect the universal scaling properties in the real-time driven process from the imaginary-time dynamics, which can be simulated via the QMC method [78,80,95–98]. In the following, we use determinant QMC (see Supplemental Material [104]) to simulate the dynamics of model (1) obeying the imaginary-time Schrödinger equation $-\frac{\partial}{\partial\tau}|\psi(\tau)\rangle = H(\tau)|\psi(\tau)\rangle$, in which g, defined as $g\equiv U-U_c$, varies linearly with the imaginary time τ as $g=\pm R\tau$ with R being the driving rate to cross the critical point U_c.

Moreover, although the original KZM focuses on scaling of topological defects generated after the quench [33,34,41,42], more general scaling behaviors exist for other quantities in the whole driven process [68–73]. By identifying an FTS region controlled by the driving-induced timescale $\zeta_R\propto R^{-z/r}$, in which z is the dynamic exponent and $r=z+1/\nu$ with ν being the correlation length exponent, the FTS

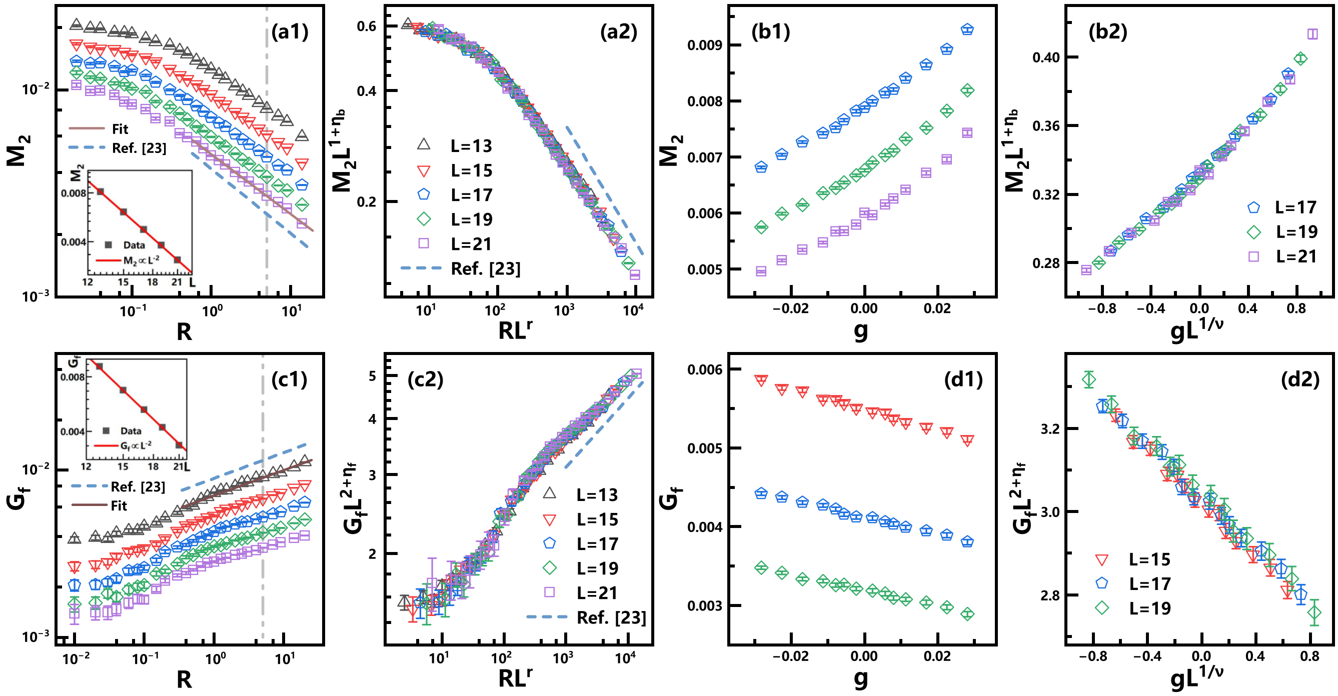

FIG. 2. Driven dynamics from the DSM phase. (a) Log-log plots of M_2 vs R driven to U_c before (a1) and after (a2) rescaling. The inset in (a1) shows $M_2 \propto L^{-2}$ at $R = 5$ (dashed-dotted line). For large R, power fitting for $L = 21$ (brown solid line) shows $M_2 \propto R^{-0.285(3)}$ with the exponent close to $(1 + \eta_b - d)/r = -0.312$ (dashed line) from Ref. [23]. (b) Curves of M_2 vs g for fixed $RL^r = 347.5$ and different L before (b1) and after (b2) rescaling. (c) Log-log plots of G_f vs R driven to U_c before (c1) and after (c2) rescaling. The inset in (c1) shows $G_f \propto L^{-2}$ at $R = 5$ (dashed-dotted line). For large R, power fitting for $L = 13$ (brown solid line) shows $G_f \propto R^{0.152(4)}$ with the exponent close to $\eta_f/r = 0.154$ (dashed line) from Ref. [23]. (d) Curves of G_f vs g for fixed $RL^r = 347.5$ and different L before (d1) and (d2) after rescaling.

theory provides the full scaling forms characterizing the nonequilibrium dynamics near the critical point [74–76], as shown in Fig. 1. Recently, it has been shown that FTS forms are still applicable in Dirac systems with short-range hopping [78]. Here, we explore the driven dynamics in the critical point with SUSY for SLAC fermions with long-range hopping.

Dynamics with DSM initial state. First, we study the driven dynamics by increasing U linearly starting from the DSM initial state, as illustrated in Fig. 1. We begin with the dynamics of the square of the SC order parameter, defined as $M_2 \equiv \frac{1}{L^4}\sum_{ij}\langle\Delta_i^\dagger\Delta_j\rangle$ with $\Delta_i = c_{i\downarrow}c_{i\uparrow}$ the on-site singlet pairing [23]. In the DSM state, it is straightforward to show that $M_2 \propto L^{-d}$. At QCP, M_2 obeys the scaling $M_2 \propto L^{-1-\eta_b}$ at equilibrium. For large R, this initial state property can be reflected at U_c, dictating that M_2 obeys

$$M_2 \propto L^{-d}R^{\frac{1+\eta_b-d}{r}}, \tag{2}$$

according to the scaling analyses. Figure 2(a) shows the dependence of M_2 on R for different L at U_c. Power fitting demonstrates that for large R, M_2 obeys a power function on R as $M_2 \propto R^{-0.285(3)}$, in which the exponent is close to $\frac{1+\eta_b-d}{r}$ with $\eta_b = \frac{1}{3}$, $\nu = 0.87$, and $d = 2$ set as the input. Moreover, with a fixed large R, Fig. 2(a1) shows that $M_2 \propto L^{-2}$. These results confirm Eq. (2). (The dataset is available in Ref. [105].)

Remarkably, the critical exponent of R in Eq. (2) is a composite of the equilibrium exponents featuring the SUSY property, demonstrating that the SUSY can manifest itself via the scaling relation of KZM and FTS. In addition, for small R, M_2 almost saturates to its equilibrium value independent of R, satisfying $M_2 \propto L^{-1-\eta_b}$ [23]. Combing the cases for large R and small R, the FTS form for M_2 at U_c should be $M_2 = L^{-1-\eta_b}\mathcal{F}(RL^r)$, which is verified via the scaling collapse in Fig. 2(a2) by substituting $\eta_b = \frac{1}{3}$ and $\nu = 0.87$. For large R, $\mathcal{F}(RL^r) \propto (RL^r)^{\frac{1+\eta_b-d}{r}}$, as shown in Fig. 2(a2), presenting the dynamic information of SUSY again. In addition, Eq. (2) is similar to the FTS relation in the pure boson model [75,77], demonstrating the universality of KZM critical scaling behavior, regardless of the presence of a gapless Dirac fermion [78].

Moreover, we show that critical properties with SUSY can be reflected via the FTS in the driven process. In this case, the full scaling form satisfies

$$M_2(R, L, g) = L^{-d}R^{(1+\eta_b-d)/r}\mathcal{F}_1(RL^r, gL^{1/\nu}), \tag{3}$$

in which $gL^{1/\nu}$ is included to take account of the off-critical-point effects. By substituting the critical exponents with SUSY, we verify Eq. (3) in Figs. 2(b1) and 2(b2). We note that Eq. (3) is consistent with the FTS in conventional bosonic QCP [75,77], and thus demonstrates its universality.

To further unravel the dynamic scaling with emergent SUSY, it is also instructive to explore the correlation function of the fermion operator, which serves as the SUSY partner of Δ_i. The equilibrium finite-size scaling of the fermion correlation $G_f(L)$, defined as $G_f(L) \equiv \frac{1}{L^2}\sum_i\langle c_i^\dagger c_{i+r_m} + \text{H.c.}\rangle$ with $r_m = (\frac{L-1}{2}, \frac{L-1}{2})$, is $G_f(L) \propto L^{-d-z+1-\eta_f}$, where $\eta_f = \frac{1}{3}$ is dictated by SUSY. When driven from the gapless Dirac phase with large R, it is expected that the initial information

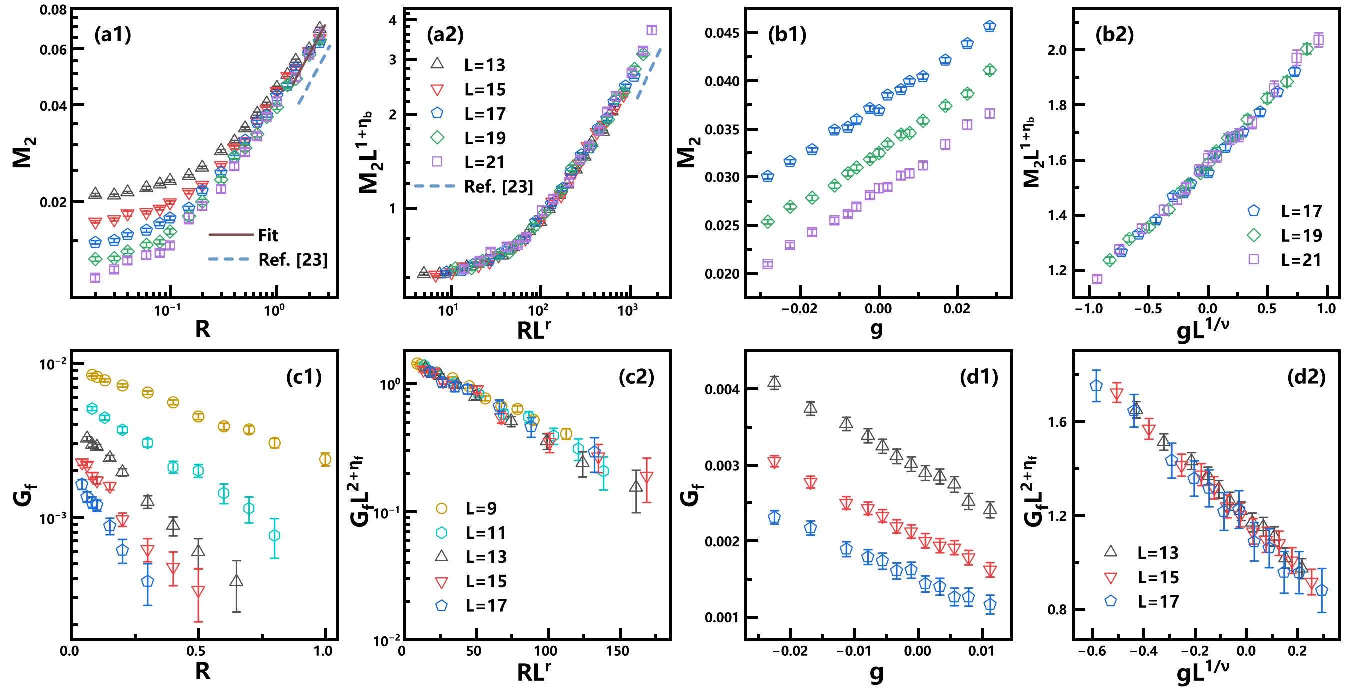

FIG. 3. Driven dynamics from the SC phase. (a) Log-log plots of M_2 vs R driven to U_c before (a1) and after (a2) rescaling. For large R, power fitting for $L = 21$ (brown solid line) shows $M_2 \propto R^{0.61(3)}$ with the exponent close to $(1 + \eta_b)/r = 0.62$ (dashed line) from Ref. [23]. (b) Curves of M_2 vs g for fixed $RL^r = 347.5$ and different L before (b1) and (b2) after rescaling. (c) Semilog plots of G_f vs R driven to U_c before (c1) and after (c2) rescaling, where the appearance of straight lines indicates the presence of exponential decay. (d) Curves of G_f vs g for fixed $RL^r = 20.85$ and different L before (d1) and (d2) after rescaling.

$G_f(L) \propto L^{-d-z+1}$ of the DSM phase should still be remembered at the critical point U_c. Accordingly, for large R, the scaling relation between $G_f(L)$ and R at U_c should be

$$G_f(R, L) = L^{-d-z+1} R^{\frac{\eta_f}{r}}. \tag{4}$$

Figure 2(c) shows the numerical results of $G_f(R, L)$. We find that for large R, $G_f(R, L)$ is a power function of R with the exponent almost independent of L. Power fitting shows that the exponent is 0.152(4), which is consistent with the results $\frac{\eta_f}{r} = 0.154$ within the error bar, given by the critical exponents for $\mathcal{N} = 2$ SUSY in 2+1 dimensions [23]. Moreover, for fixed R, Fig. 2(c1) shows that $G_f(R, L) \propto L^{-2}$. These results confirm Eq. (4). Similar to Eq. (2), Eq. (4) demonstrated that the SUSY can be delivered in the scaling of R. Remarkably, we also obtain the scaling relation of the fermion correlator for the driven dynamics with the DSM initial state, as dictated in Eq. (4), which can be generalized to other Dirac fermionic QCPs.

In addition, combining the scaling relation with large and small R, one can obtain the FTS form $G_f = L^{-d-z+1-\eta_f}\mathcal{G}(RL^r)$ in which $\mathcal{G}(RL^r) \propto (RL^r)^{\frac{\eta_f}{r}}$ for large R. These results are confirmed in Fig. 2(c2). Moreover, similar to Eq. (3), we show that although SUSY emerges exactly at U_c, it can affect the dynamic scaling of G_f in the driven process. With $gL^{1/\nu}$ included, the FTS form can be generalized as

$$G_f(R, L, g) = L^{-d-z+1} R^{\eta_f/r} \mathcal{G}_1(RL^r, gL^{1/\nu}). \tag{5}$$

For an arbitrarily fixed RL^r, we verify Eq. (5) in Figs. 2(d1) and 2(d2) by substituting the critical exponents with SUSY.

Dynamics with SC initial state. We then explore the dynamic scaling with emergent SUSY from the SC initial state. In Fig. 3(a1), we find that for large R, M_2 increases as $M_2 \propto R^{0.61(3)}$ with the exponent close to $\frac{1+\eta_b}{r}$ and almost does not depend on L, demonstrating that

$$M_2 \propto R^{\frac{1+\eta_b}{r}}. \tag{6}$$

Here, the anomalous dimension $\eta_b = \frac{1}{3}$ again enters the scaling relation depending on R as a signature of SUSY. Combining these results with the finite-size scaling for small R, one obtains the FTS form at U_c as $M_2 = R^{\frac{1+\eta_b}{r}} \mathcal{F}_2(RL^r)$, which is verified in Fig. 3(a2).

By including $gL^{1/\nu}$, the full FTS form of M_2 in the driven process reads [68,75,78,79]

$$M_2(R, L, g) = R^{\frac{1+\eta_b}{r}} \mathcal{F}_3(RL^r, gL^{1/\nu}), \tag{7}$$

which is verified in Figs. 3(b1) and 3(b2), demonstrating that the scaling with SUSY can appear in the driven process near the critical point.

Then, we explore the dynamic scaling of the fermion correlation $G_f(L)$. For large R, we find that G_f at U_c changes with R as an exponentially decaying function as shown in Fig. 3(c1). This is because for larger R, information in the gapped SC phase can be brought to U_c. By rescaling G_f and R by L according to the critical exponents with SUSY, we find that the rescaled curves collapse onto each other, as shown in Fig. 3(c2), confirming the FTS form of $G_f(L)$ at U_c is $G_f = L^{-2-\eta_f}\mathcal{G}_2(RL^r)$. In addition, Fig. 3(c2) also shows that the scaling function $\mathcal{G}_2(RL^r)$ satisfies $\mathcal{G}_2(RL^r) \propto \exp(-RL^r)$.

Moreover, we show that SUSY can affect the dynamic scaling of $G_{\rm f}$ in the driven process. With $gL^{1/\nu}$ included, the FTS form can be generalized as

$$G_{\rm f}(R, L, g) = L^{-2-\eta_{\rm f}} \mathcal{G}_3(RL^r, gL^{1/\nu}). \quad (8)$$

For an arbitrarily fixed RL^r, we verify Eq. (8) in Figs. 3(d1) and 3(d2) by substituting the critical exponents with SUSY.

The hallmark of emergent SUSY in nonequilibrium dynamics. The hallmark of emergent SUSY is the equivalence of anomalous dimensions for boson and fermion. In contrast to previous sections, in which critical exponents are set as the input to verify the scaling theory, here we show that the dynamic scaling theory for large R provides an efficient way to determine critical exponents with high accuracy. According to Eqs. (2) and (6) and the results of M_2 vs R shown in Figs. 2(a1) and 3(a1), we access boson anomalous dimensions $\eta_b = 0.36(4)$ and $r = 2.23(8)$. Similarly, from Eq. (4) and the result in Fig. 2(c1), the fermion anomalous dimension is achieved, $\eta_f = 0.337(14)$. The values of boson and fermion anomalous dimensions are identical to each other and consistent with exact values $\eta_b = \eta_f = \frac{1}{3}$ within the error bar, providing convincing evidence of emergent SUSY. Hence, our results unambiguously demonstrate that the nonequilibrium dynamic scaling is a powerful approach to reveal the feature of emergent SUSY, offering a practical tool to detect emergent SUSY in experimental platforms.

Summary. In summary, we study the driven dynamics of a QCP with emergent SUSY, through sign-problem-free QMC simulation. Driving the system from both DSM and SC phases, we discover interesting nonequilibrium scaling behaviors. In particular, we unveil that SUSY can manifest itself in the scaling relations of KZM and FTS. Moreover, we show that SUSY also plays roles in the driving process by obeying the FTS forms with the critical exponents of the SUSY critical point. The scaling relation of the fermion correlation on the driving rate is obtained, which can be generalized to other Dirac fermionic QCPs.

Recently, programmable quantum processors have been developed as advanced platforms to realize different phases. In these experiments, the KZM and the FTS are generally used in state preparation. The dynamic scaling for the QCP with emergent SUSY revealed here provides a practical theoretical framework to detect emergent SUSY in experimental platforms. Noting the progress on the experimental proposal for $(1+1)$D SUSY [27], it is expected that our present results are detectable and potentially helpful for investigating QCP with emergent SUSY in these systems. In addition, our results unambiguously demonstrate that SLAC fermions with long-range hopping still satisfy the nonequilibrium scaling of KZM, although it was shown that special caution should be paid to the unexpected finite-temperature transition related to the excited states [106].

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Data availability. The data that support the findings of this article are openly available [105].

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