课程安排　　余蔚锐 — 第一章　　其他 — trivial

莫梁灯 — 最后一章

前置基础　　微积分 + 零物理基础

第一章

一个虚拟的世界

一些预设

质点　　定义　描述其运动时可忽略大小的物体

质点状态　　位置　q

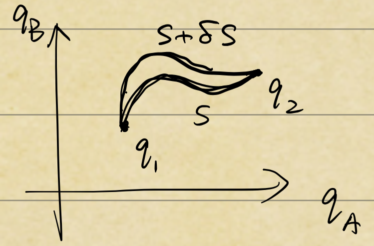
速度　$\dot{q} = v$

描述质点　　$L(q, \dot{q}, t)$

描述运动　　$S = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$

规律　　最小作用量原理：从 q_1 到 q_2 的运动满足"S最小"

一个工具



$$\delta S = 0$$

$$= \delta \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$$

$$= \int_{t_1}^{t_2} \frac{\partial L}{\partial q} \delta q\, dt + \int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}}\, d(\delta q)$$

$$= \cancel{\frac{\partial L}{\partial \dot q}\delta q\Big|_{t_1}^{t_2}} + \int_{t_1}^{t_2}\left[\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)\right]\delta q\, dt = 0$$

拉格朗日方程 $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L}{\partial q} = 0$

$\downarrow$

L 不唯一

开始简化

$$L'(q,\dot q,t) = L(q,\dot q,t) + \frac{d}{dt}f(q,t)$$

惯性定律 定义惯性系：时空均匀的，各向同性的

$L(v^2)$

$\frac{\partial L}{\partial r} = 0 = \frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) = 0$

$\frac{\partial L}{\partial v} = \text{const}$

$v = \text{const}$

质量 两惯性系 $v' = v + \varepsilon$ 小量

$L = L(v^2)$

$L' = L'(v'^2) = L(v^2) + \underline{2\frac{\partial L}{\partial (v^2)} v\cdot\varepsilon}$

$2\frac{\partial L}{\partial (v^2)} = \text{const} = m$

$L = \frac{1}{2}mv^2 = T$

质量可加性 两个质点系 A, B

$L_A + L_B = L$

$\Rightarrow$ m 可加性 $\rightarrow$ 广延性

质量非负性 $\underline{m<0}$ $L = \frac{1}{2}mv^2 < 0$

$S = \int_{t_1}^{t_2} L\, dt \rightarrow -\infty$ 无最小值 ✗

牛顿第二定律　封闭质点系

$$L = \sum_a \frac{1}{2} m_a v_a^2 - U(r_1, r_2, \cdots)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

$$m_a \frac{d}{dt} v_a = -\frac{\partial U}{\partial r_a}$$

力

$$F_a = -\frac{\partial U}{\partial r_a}$$

能量/能量守恒　封闭质点系

运动积分 $2s-1$ 个

时间均匀性

$$\frac{dL}{dt} = \sum_i \frac{\partial L}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i$$

$$= \sum_i \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i$$

$$= \sum_i \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i\right)$$

$$\frac{d}{dt}\left(\sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L\right) = 0$$

$$\begin{cases} \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L = \text{const} = E \\ L = T - U \\ \sum_i \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i = 2L \end{cases}$$

$$\Rightarrow \quad E = T(q, \dot{q}) + U(q)$$

动量/守恒　空间均匀性

$$r_a \to r_a + \varepsilon$$

→小量

$$\delta L = \sum_a \frac{\partial L}{\partial r_a}\delta r_a = \varepsilon \cdot \sum_a \frac{\partial L}{\partial r_a} = 0$$

$$\sum_a \frac{\partial L}{\partial r_a} = 0$$

$$\frac{d}{dt}\left(\sum_a \frac{\partial L}{\partial v_a}\right) = 0$$

$$\sum_a m_a v_a = P$$